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A new model of fatigue damage and its experimental verification[1]

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Abstract. Based on the classical thermodynamic theory, the internal material damage evolution process can be considered as a result of the increasing of dissipative potential. Taking Q235 steel as an example, the tensile tests of repeated loading and unloading of non-symmetrical cycle on the CMT5105 universal electronic testing machine were conducted. From the view point of thermal dissipation, with the combination of experimental results, a new fatigue damage model is derived by using the internal variable theory and the generalized orthogonal principle. The results show that the new fatigue damage model can satisfy the classical law of thermodynamics and reflect the energy dissipation behavior of the internal variable with the advantages of clear physical meaning and simple form as well as few parameters. Also, it coincides well with the experimental results.

Key words. Fatigue damage model, dissipation potential, thermodynamics,, strain rate..

1. Introduction

Defects inside the material, such as dislocation, micro cracks, holes and so on, is called damage, the dissipation process of deterioration of material damage is irreversible [1]. The defects of the internal distribution of the material under the combined action of external load and environmental factors is continued to evolve, which would finally lead to the destruction of the material. The classical thermody-

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namic theory, based on the Helmholtz specific free energy, is used as the constitutive function in the thermodynamic state space of the damage material. By means of a series of forms in the thermodynamic state space, the evolution relation and constitutive equation of damage have been established [2]. However, most of the constitutive equations are complicated and need more parameters, which is not convenient for practical application. At the same time, some relatively simple constitutive equations lack sufficient theoretical basis, and the physical meaning is not clear enough.

Based on classical thermodynamic theory, an effective model of fatigue damage evolution is proposed based on the studying of the fatigue damage of materials via dissipative potential, and the fatigue damage model could be verified by the test data and the results were obtained. Meanwhile, the feasibility of the fatigue damage model is confirmed compared with other experimental results by highlighting the strain rate.

2. Fatigue damage evolution model

2.1. Basic equations of irreversible thermodynamics

The movement and state change of matter in nature can be regarded as a thermodynamic process, which can be divided into reversible thermodynamic process and irreversible one. According to the definition, damage is an irreversible thermodynamic process, under the framework of thermodynamics, the second law of thermodynamics should be satisfied. An important concept - entropy is related to the second law of thermodynamics. Entropy is a kind of thermodynamic state parameter which is only decided by the state. The numerical value of this state function can be used to determine the thermodynamic process. The equilibrium state entropy is defined as

$$\mathrm{d}s = \frac{\mathrm{d}Q}{T}\,, \tag{1}$$

where s is the equilibrium entropy, Q is the heat exchange in thermodynamic process and T is temperature.

Describe by the entropy, the second law of thermodynamics states that the entropy never decrease during the material system alters from one equilibrium state to another. If the process is irreversible, the entropy will be increased. Since the material damage is an irreversible process, it should obey the second law of thermodynamics, the practical Clausius-Duhem inequality is expressed as follows [3]

$$\int_V \rho\dot{s}\,\mathrm{d}V + \int_{\partial V} \frac{\dot{q}_i}{T}\,\mathrm{d}A_i > 0\,, \tag{2}$$

where $\dot{s}$ is the equilibrium entropy rate, the formula is rewritten by using the Gauss integral and then conduct to combine it.

$$\int_V \rho\dot{s}\,\mathrm{d}V + \int_V \frac{\dot{q}_{i,j}}{T}\,\mathrm{d}V - \int_V \frac{\dot{q}_j T_i}{T^2}\,\mathrm{d}V > 0\,, \tag{3}$$

T_i is absolute temperature T along the direction $i\,(i = x,\, y,\, z)$ of the gradient of temperature, so the inequality of damaged element can be acquired to satisfy the second law of thermodynamics

$$\rho \dot{s} T + \dot{q}_{i,j} - \frac{\dot{q}_j T_i}{T} > 0 \,. \tag{4}$$

Using the ability of Helmholtz free energy characterization system for external work, there is

$$g = e - Ts \,, \tag{5}$$

where g is the free energy density, e is the internal energy of the unit mass of the damaged medium. Taking the free energy into equation (4) is

$$\rho \dot{e} - \rho \dot{g} - \rho \dot{T} s + \dot{q}_{i,j} - \frac{\dot{q}_j T_i}{T} > 0 \,. \tag{6}$$

Here, the material derivative is expressed by the dots above the variable.

The form of rate is rewritten by the law of conservation of energy, there is

$$\rho \dot{e} + \dot{q}_{i,j} = \sigma_{ij} \dot{\varepsilon}_{ij} \,, \tag{7}$$

where σ_{ij}, ε_{ij} are expressed as the Cauchy stress and infinitesimal strain tensor, respectively, taking (**??**) into formula (6) we can get

$$\sigma_{ij} \dot{\varepsilon}_{ij} - \rho \dot{g} - \rho \dot{T} s - T^{-1} \dot{q}_j T_i > 0 \,. \tag{8}$$

For an isothermal infinitesimal deformation process, the inequality produced by the local entropy of material is [4]

$$\sigma_{ij} \dot{\varepsilon}_{ij} - \dot{\psi} \geq 0 \,, \tag{9}$$

which ψ is the unit volume ratio of Helmholtz free energy, the subscript should obey the convention of summation.

It is assumed that the material is elastic and isotropic in the process of damage, and the Helmholtz ratio of free energy ψ is related to the strain ε and damage variable D, i.e.

$$\psi = \psi(\varepsilon_{ij}, D) \,. \tag{10}$$

Differential can be obtained from the formula (10)

$$\dot{\psi} = \frac{\partial \psi}{\partial \varepsilon_{ij}} \dot{\varepsilon}_{ij} + \frac{\partial \psi}{\partial D} \dot{D} \,. \tag{11}$$

Taking formula (11) into (10), it has

$$\left(\sigma_{ij} - \frac{\partial \psi}{\partial \varepsilon_{ij}} \right) \dot{\varepsilon}_{ij} - \frac{\partial \psi}{\partial D} \dot{D} \geq 0 \,. \tag{12}$$

For arbitrary values the inequality (12) of $\dot{\varepsilon}_{ij}$ is established, which shows that

$$\sigma_{ij} = \frac{\partial \psi}{\partial \varepsilon_{ij}} \,. \tag{13}$$

Order $Y = -\partial\psi/\partial D$, by the formula (12), (13) we can get

$$Y\dot{D} \geq 0 \,. \tag{14}$$

Y is expressed as generalized thermodynamic force and coupling representation of damage variable D, which called the release rate of damage strain energy. Formula (13) is the stress-strain elastic constitutive law of the damage material, and the formula (14) is the damage strain energy release rate , i.e. the damage constitutive law, $Y\dot{D}$ is the damage dissipative power in the process of damage. The formula (14) shows that the damage process is an energy dissipative process, the dissipative properties can be described by another thermodynamic potential ψ^*, which called dissipative potential or damage flow potential, it is a convex function of Y [5]–[6].

The general form of damage evolution equation can be obtained by the orthogonal flow rule of internal variable

$$\dot{D} = \partial\psi^*/\partial Y \,. \tag{15}$$

By formula (15) it is known that the damage evolution model can be obtained if the state potential ψ and the damage flow potential $\psi^*$are given.

2.2. Dissipative potential and damage evolution equation

Dissipative potential ψ^* is expressed as the power function of Y by reference [7], that is

$$\psi^* = \frac{1}{m+1} A Y^{m+1} \,. \tag{16}$$

Taking formula (16) into (15) is

$$\dot{D} = \partial\psi^*/\partial Y = \frac{A}{m+1} \cdot (m+1) Y^m \cdot \dot{Y}/\partial Y = A Y^m \,. \tag{17}$$

A, m are non negative parameters to characterize the damage evolution of material, which can be determined by the test data.

Referring to the fatigue damage model by scholars [8], [9]. In general, the fatigue damage equation of the metal material can be expressed as

$$\frac{\mathrm{d}D}{\mathrm{d}N} = H \exp\left[-\frac{U_{\mathrm{S}}(\sigma_\alpha,\, \sigma_{\mathrm{m}},\, D)}{kT}\right] \tag{18}$$

In the formula, $U_{\mathrm{S}}(\sigma_\alpha, \sigma_{\mathrm{m}}, D)$ is defined as heat dissipation energy, σ_α and σ_{m} are cyclic amplitude of stress and mean stress, respectively. D is defined as damage variable for the current damaged state, H is a material constant.

When the temperature is constant, the formula (18) can be written as

$$\frac{\mathrm{d}D}{\mathrm{d}N} = H \exp[-G(\sigma_\alpha,\, \sigma_{\mathrm{m}},\, D)]\,. \tag{19}$$

The following model [10], [11] was proposed by Chaboche (1974)

$$\frac{\mathrm{d}D}{\mathrm{d}N} = \left[1-(1-D)^{B+1}\right]^{A(\sigma_{\mathrm{M}},\,\bar{\sigma})} \left[\frac{\sigma_{\mathrm{M}}-\bar{\sigma}}{M(\bar{\sigma})(1-D)}\right]^{B}. \tag{20}$$

In the formula, σ_{M} is the maximum stress, $\bar{\sigma}$ is the average stress. Coefficients B and functions $A(\sigma_{\mathrm{M}}, \bar{\sigma})$ are related to the material and temperature.

In the formula (20), index B is assumed to be a constant, for some materials, the load stress ratio is significantly correlated with the index B, so Wang proposed a new damage model (Wang 1992a) [12]

$$\frac{\mathrm{d}D}{\mathrm{d}N} = \left[\frac{\alpha\sigma_\alpha + (1-\alpha)\,\bar{\sigma}}{M(R)(1-D)}\right]^{n(R)}. \tag{21}$$

In the formula, $\sigma_\alpha = \sigma_{\mathrm{M}} - \bar{\sigma}$ is the stress amplitude, $R = \sigma_{\mathrm{m}}/\sigma_{\mathrm{M}}$ is the stress ratio, coefficient α and function $M(R)N(R)$ are related to material and temperature.

It is assumed that the initial damage of the material $D_0 = 0$, the fracture damage value $D_{\mathrm{c}} = 1$, the fracture life of the material can be obtained by integrating formula (21)

$$N_{\mathrm{f}} = \frac{1}{n(R)+1}\left[\frac{\alpha\sigma_\alpha + (1-\alpha)\bar{\sigma}}{M(R)}\right]^{-n(R)}. \tag{22}$$

The damage value at any time can be expressed as

$$D = 1 - \left[1 - \frac{N}{N_{\mathrm{f}}}\right]^{\frac{1}{n(R)+1}}. \tag{23}$$

The rate of damage change from different n values can be seen: when the n value is larger, the damage evolution is accelerated sharply when the material is close to fracture, and to the smaller n value, the damage evolution is gradually changed.

Based on the data from the fatigue test, the plot is drawn by the related parameters of fatigue damage evolution model from ref. [13], then according to the characteristics of damage evolution, as shown in Fig. 1, the more realistic damage evolution law can be given.

Referring to Fig. 1, on the basis of formula (17), the power function linear form could be used to express the fatigue damage evolution law

$$D = a\kappa^m x^b + cx\,, \tag{24}$$

where κ is the ratio of stress amplitude and maximum stress, that is $\kappa = \frac{\sigma_\alpha}{\sigma_{\max}}$; D is a damage factor; $x = N/N_{\mathrm{f}}$, N is represented as the number of cycles, and N_{f} is the number of cycles when the complete failure occurred (related with cyclic stress level).

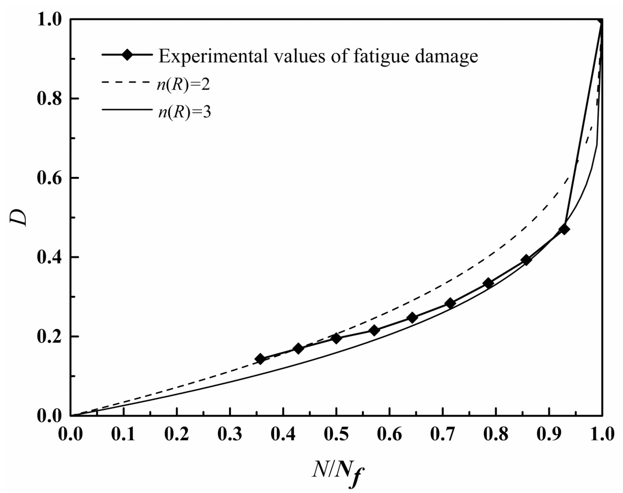

Fig. 1. Comparison among the experimental curve between fatigue damage - versus lifetime and the curves of literature models

If the loading is not in the yield stage, the material is not damaged, that is, when $x = N/N_{\mathrm{f}} = 0$, $D = 0$, it can be showed that formula (24) is effective. And when the material is completely destroyed, $x = N/N_{\mathrm{f}} = 1$, $D = 1$, taking to the formula (24) it has $c = 1 - a\kappa^m$. κ can be calculated by the test data, the index m is represented as the degree of damage related factors. Thus, the value of c depends on the value of ma. Therefore, the formula (24) has only 2 independent parameters need to be determined.

3. Case analysis

3.1. Experimental equipment

The test material is Q235 steel, and the fatigue test specimen is shown in Fig. 2.

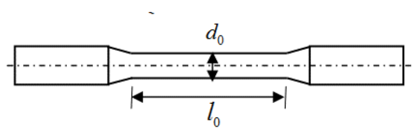

Fig. 2. Schematic diagram of standard test specimen

In Fig. 2, d_0 is the diameter, l_0 is the standard fatigue test sample length. $d_0 = 10\,\mathrm{mm}$, $l_0 = 50\,\mathrm{mm}$.

The testing of the tension and compression of non-symmetrical cycle was carried out on the CMT5105 universal electronic testing machine by the loaded rate of 2 mm/min, 3 mm/min, 4 mm/min, respectively.

3.2. Experimental results

In the fatigue test of load control, according to the theory of elastic plastic damage [14], the ratio of residual strain and final strain was processed after comparison [15] and then combined with the change of the cyclic stress amplitude in the process of non symmetrical cycle, the damage amount can be obtained under a certain cycle, as shown in Table 1.

3.3. Parameter determination

By using the test data, the value of κ can be calculated

$$\kappa = \frac{\sigma_\alpha}{\sigma_{\max}} = \frac{202.2975}{404.7423} = 0.499818\,.$$

Taking it into the formula (24), the relationship curve of damage and life of 3 groups could be drawn when a, b, m takes different values, then other two curves are drawn according to the literature [13] and test results, as shown in Fig. 3. The results showed that, by adjusting the value of a, b, m, The formula (24) is in good agreement with the experimental curve and is more closer to the test value than the theoretical value of literature [13].

Thus, the fatigue damage evolution law can be expressed as formula (24) or

$$x = \sqrt[b]{\frac{D}{a\kappa^m}}\left(\sqrt{D} + 1.5cD - eD^2\right)\,. \tag{25}$$

By using (19) and (24) we can get

$$\frac{\mathrm{d}D}{\mathrm{d}N} = H\exp[-G(\sigma_\alpha,\,\sigma_\mathrm{m},\,D)] = \frac{1}{N_\mathrm{f}}\left[ab\kappa^m x^{b-1} + c\right]\,. \tag{26}$$

Substituting Eq. (25) into Eq. (26) it can be obtained

$$\frac{\mathrm{d}D}{\mathrm{d}N} = H\exp[-G(\sigma_\alpha,\,\sigma_\mathrm{m},\,D)] =$$

$$= \frac{1}{N_\mathrm{f}}\left\{ab\kappa^m\left[\sqrt[b]{\frac{D}{a\kappa^m}}\left(\sqrt{D} + 1.5cD - eD^2\right)\right]^{b-1} + c\right\}\,. \tag{27}$$

When $D = 1$, the critical dissipative potential under cyclic loading can be seen as a complete dissipation, thus, the formula (27) can be transformed into

$$H = \frac{1}{N_\mathrm{f}}\left\{ab\kappa^m\left[\sqrt[b]{\frac{1}{a\kappa^m}}\,(1 + 1.5c - e)\right]^{b-1} + c\right\}\,. \tag{28}$$

The expression of $G(D)$ can be obtained by the formula (28)

$$G(D) = -\ln\left[\frac{g(D)}{H}\right],$$

$$g(D) = \frac{1}{N_{\mathrm{f}}}\left\{ab\kappa^{m}\left[\sqrt[b]{\frac{D}{a\kappa^{m}}}\left(\sqrt{D} + 1.5cD - eD^{2}\right)\right]^{b-1} + c\right\}. \tag{29}$$

Here $\kappa = 0.499818$, $c = 1 - a\kappa^{m}$. Only the a, b, m, e four parameters need to be established. Using a to represent the initial damage evolution rate, m indicates the degree of the mutual influence between the damage, e indicates the actual damage of the material parameter correction value. Combining Fig. 2 and Fig. 3, it could be seen that a large number of theoretical images can be obtained in accordance with the experimental data via the adjustment of the value of a, b, m, e.

Table 1. The test values of D and N/N_{f}

number	N/N_{f}	D
1	0.357143	0.143341
2	0.428571	0.169591
3	0.5	0.195314
4	0.571429	0.215386
5	0.642857	0.247353
6	0.714286	0.283673
7	0.785714	0.33411
8	0.857143	0.392829
9	0.928571	0.470504
10	1	1

3.4. Comparative analysis

According to Fig. 3, it can be seen that through the adjustment of the value of a, c, b, m, the relative rate of the damage life relationship can be established. Also seen from Fig. 3, when the speed loading problem is in the condition of low rate, the error between the theoretical and experimental values is tiny and the curves are almost overlapped, at this point the impact of the rate factors can be ignored by the material damage. With the gradual increasing of the rate, the damage calculated by the fatigue damage model is closer to the experimental value when the damage threshold is finally reached.

Through the new fatigue damage model, the damage state of metal materials can

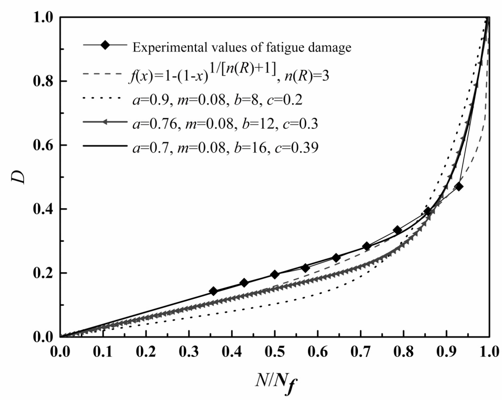

Fig. 3. Comparison among the theoretical curves between fatigue damage - versus lifetime based on equation (24) and the experimental curve

be effectively predicted, and the fatigue failure in the project can be prevented to a certain extent.

3.5. Damage evolution model with consideration of velocity under isothermal and cyclic stress

In order to further study the influence of the rate of damage evolution model, several groups of different constant speeds of fatigue damage experiment has been conducted. At the same time, the fatigue damage evolution law under the isothermal and cyclic stress amplitude is also expressed in the linear form of power function

$$D = a\kappa^m x^b + cx\,. \tag{30}$$

According to Eq.(30), the relationship among several values of a, c, b, m of damage and life is shown in Fig. 4, compared with the experimental results (Specific data are shown in Table 2, 3, 4), it can be seen that through the adjustment of a, c, b, m values, the optimum damage life relationship relative to its speed can be obtained. And the previous conclusions has been synthesized and corrected, a represents the initial damage evolution rate, m is determined by the material characteristics, b is related to the level of cyclic stress and load levels, and c represents the degree of interaction between the damage factors. As seen from Fig. 4, with the gradual increase of the rate, the error between the theoretical value and the experimental value is gradually reduced, and the accuracy of the image matching has improved.

From Fig. 4, we can basically establish the relationship between the coefficients of Eq. (24), take the rate as the key point, the formula (24) can be rewritten as

$$D = a_0\left(1 + 0.02\cdot\frac{\Delta\nu}{\nu_0}\right)\kappa^{m_0}x^{\left(b_0 + d\cdot\frac{v}{\Delta\nu}\right)} + \left[c_0 + 0.01\cdot\frac{\Delta\nu}{\nu_0}\right]x\,. \tag{31}$$

Table 2. The test values of D and N/N_f about experiment 1 data of 2 mm/min

number	N/N_f	D
1	0.533333	0.138277
2	0.6	0.160017
3	0.666667	0.186504
4	0.733333	0.219390
5	0.8	0.262975
6	0.866667	0.328553
7	0.933333	0.473242
8	1	1

Table 3. The test values of D and N/N_f about experiment 1 data of 4 mm/min

number	N/N_f	D
1	0.733333	0.203462
2	0.8	0.243106
3	0.866667	0.297895
4	0.933333	0.390980
5	1	1

Table 4. The test values of D and N/N_f about experiment 1 data of 6 mm/min

number	N/N_f	D
1	0.733333	0.209406
2	0.8	0.246868
3	0.866667	0.297168
4	0.933333	0.376683
5	1	1

Here, parameter $d = \frac{2\Delta v}{v_0}$, ν_0 is the initial rate, $\Delta\nu$ is the increment of rate. Considering the initial situation, $\Delta\nu = 0$, $b = b_0$ is coincide with the actual situation. When $\Delta\nu \neq 0$, $b = b_0 + \frac{2\Delta v}{v_0} \cdot \frac{v}{\Delta v} = b_0 + \frac{2v}{v_0}$, substituting the data is feasible. As a result, it can be simply obtained the fatigue damage evolution equation of uniform loading under constant temperature and cyclic stress.

To compare the differences between constant and variable speeds loading, we used the previous test data to analysis as shown in Fig. 5.

From Fig. 5, the curves of group 1 and group 2, 3, 4 are almost parallel, which shows that there is a similar trend between them. And the curves of group 2, 3, 4 were further observed, we can see that when approaching the critical value of the injury, the trend of image changes suddenly increases with the rate rising, and the corresponding cycle index will also decrease. This shows that when the material reaches the failure limit, the effect of strain rate on fatigue damage of materials is also increased, and the higher strain rate will lead to premature failure of the material.

In addition, the observed images show that the evolution process of variable fatigue damage is more complicated, but the damage life curve of the fitting test results can be obtained by adjusting the value of a, c, b, m, which is shown that the new fatigue damage model is applicable to both constant and variable strain rate conditions.

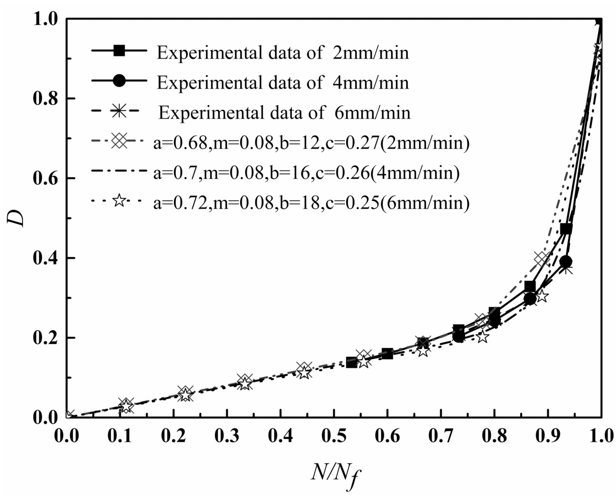

Fig. 4. Comparison among the theoretical curves between fatigue damage - versus lifetime based on equation (24) and the experimental curve

4. Conclusions

1. The process of fatigue damage evolution can be seen as the process of energy dissipation, and its dissipation characteristics can be described by another thermodynamic potential ψ^*, and the general form of damage evolution equation can be obtained by the orthogonal flow rule of internal variable.

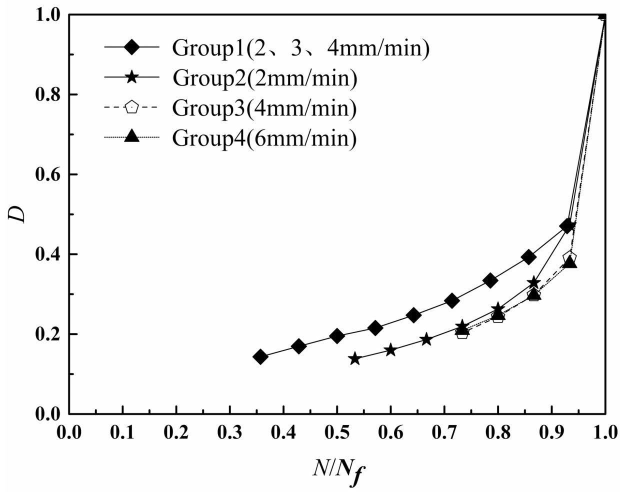

Fig. 5. Measured curves of the relationship between fatigue damage - versus lifetime of different loading rates

$$\dot{D} = \partial\psi^*/\partial Y\,.$$

Here the damage evolution model can be obtained if the state potential ψ and the damage flow potential ψ^* are given.

2. The theoretical form of fatigue damage evolution equation can be expressed as

$$\frac{\mathrm{d}D}{\mathrm{d}N} = H\exp\left[-\frac{U_{\mathrm{S}}(\sigma_\alpha,\,\sigma_{\mathrm{m}},\,D)}{kT}\right].$$

In the formula, $U_{\mathrm{S}}(\sigma_\alpha,\,\sigma_m,\,D)$ is defined as heat dissipation energy, σ_α and σ_{m} are cyclic stress amplitude and mean stress, respectively. D is defined as damage variable for the current damage state, H is material constant.

3. When the temperature is constant, the theoretical form of fatigue damage evolution equation can be written as

$$\frac{\mathrm{d}D}{\mathrm{d}N} = H\exp[-G(\sigma_\alpha,\,\sigma_{\mathrm{m}},\,D)]\,.$$

According to the characteristic of damage evolution and the combination of general form of damage evolution equation, the fatigue damage evolution law can be expressed in the linear form of power function

$$D = a\kappa^m x^b + cx\,,$$

where κ is the ratio of stress amplitude and maximum stress, that is $\kappa = \frac{\sigma_\alpha}{\sigma_{\max}}$;$D$ is damage factor; $x = N/N_{\mathrm{f}}$, N represents the number of cycles, and N_{f} is the number of cycles when complete failure occurred (related with cyclic stress level).

The above equation had been verified by experimental data, and has certain theoretical basis.

4. The damage evolution law under isothermal and cyclic stress is established. based on the rate factor, the evolutionary law has only 2 parameters to be determined, and its theoretical value had been verified by the actual data.

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